A Modified Unsplit PML Formulation for Evanescent Mode Absorption in Waveguides

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***Abstract*—In this letter, a modified unsplit perfectly matched layer (PML) formulation for the absorption of evanescent waves in waveguides is presented. The proposed formulation combines the advantages of the stretched coordinate and unsplit $D-E-H$ PML formulations, which separates PML conductivities from the properties of the physical materials in the FDTD mesh. Results show that the proposed boundary outperforms the traditional unsplit PML in the evanescent region by 20 to 40 dB.**

***Index Terms*—Evanescent waves, FDTD, unsplit PML.**

I. Introduction

WAVEGUIDE structures can be analyzed by the well known generalized scattering matrix (GSM) technique [1]. The idea behind this technique is to break the electromagnetic problem into several blocks and to solve each of these by using numerical techniques such as method of moments (MoM), finite element method (FEM), finite difference time domain (FDTD) or mode matching (MM). The procedure of cascading blocks generally resorts to the frequency domain scattering matrices of each block, which can conveniently be obtained from the frequency domain techniques (MoM, FEM, and MM). On the other hand, FDTD provides the time domain system response due to excitation waveforms with a predefined spectrum of frequency components. Thus, in order to use FDTD with frequency domain techniques in the GSM approach for waveguide problems, the frequency domain modal information has to be accurately extracted from the total fields in the time domain. For this purpose, an absorbing boundary that works for all frequencies, regardless of the mode (i.e., propagating and/or evanescent) is needed.

In this work, a new PML absorbing boundary formulation for overmoded waveguides is developed. The proposed formulation extends the traditional $D-E-H$ unsplit field PML [2] to provide superior absorption for both evanescent and propagating waves in a waveguide for all frequencies.

Manuscript received September 9, 2002; revised December 1, 2002. This work was supported by a Army Research Office-MURI grant under the Spatial and Quasi-Optical Power Combining DAAG-55-97-K-0132. The review of this letter was arranged by Associate Editor Dr. Rüdiger Vahldieck.

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Digital Object Identifier 10.1109/LMWC.2003.814109

II. Theory

Indicating the magnetic and electric conductivities with $[\sigma_M]$ and $[\sigma_E]$, respectively, and the permittivity and permeability tensors with $[\vec{\epsilon}]$ and $[\vec{\mu}]$, respectively, the complex diagonal tensors for biaxial media can be written as

$$[\vec{\epsilon}] = \epsilon_0 \begin{pmatrix} \epsilon_x + \frac{\sigma_E^x}{j\omega\epsilon_0} & 0 & 0 \\ 0 & \epsilon_y + \frac{\sigma_E^y}{j\omega\epsilon_0} & 0 \\ 0 & 0 & \epsilon_z + \frac{\sigma_E^z}{j\omega\epsilon_0} \end{pmatrix} \quad (1)$$

$$[\vec{\mu}] = \mu_0 \begin{pmatrix} \mu_x + \frac{\sigma_M^x}{j\omega\mu_0} & 0 & 0 \\ 0 & \mu_y + \frac{\sigma_M^y}{j\omega\mu_0} & 0 \\ 0 & 0 & \mu_z + \frac{\sigma_M^z}{j\omega\mu_0} \end{pmatrix} \quad (2)$$

so that Maxwell's equations can be written as

$$\nabla \times \vec{E} = -j\omega[\vec{\mu}]\vec{H} \quad (3)$$

$$\nabla \times \vec{H} = j\omega[\vec{\epsilon}]\vec{E}. \quad (4)$$

Sacks *et al.* [3] showed that the following condition is required to match the PML medium to free space:

$$\frac{[\vec{\epsilon}]}{\epsilon_0} = \frac{[\vec{\mu}]}{\mu_0}. \quad (5)$$

Accordingly, the $[\vec{\epsilon}]$ and $[\vec{\mu}]$ can be rewritten as

$$[\vec{\epsilon}] = \epsilon_0 \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix} \quad (6)$$

$$[\vec{\mu}] = \mu_0 \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}. \quad (7)$$

For absorption in the direction perpendicular to the boundary (for example z direction), the relative dielectric constant and the relative permeability must be the inverse of those in the other directions [3]

$$a = b = \frac{1}{c}. \quad (8)$$

This condition provides a perfectly reflectionless absorbing boundary for any frequency, any angle of incidence and any polarization in the z direction.

As indicated in [4], the above formulation is equivalent to the Chew–Weedon formulation [5] and therefore it is possible to modify the formulation in [6] (which uses the Chew–Weedon formulation) to a $D-E-H$ formulation of the FDTD update equations. Following the approach in [2], it is convenient to normalize the fields $\vec{D}$ and $\vec{E}$ as follows:

$$\vec{E}' = \sqrt{\frac{\epsilon_0}{\mu_0}}\,\vec{E} \tag{9}$$

$$\vec{D}' = \frac{1}{\sqrt{\epsilon_0\mu_0}}\,\vec{D}. \tag{10}$$

With the additional coordinate stretching variables $s_x = \sqrt{bc}$, $s_y = \sqrt{ca}$ and $s_z = \sqrt{ab}$, Maxwell's equations in normalized $D-E-H$ form become

$$j\omega\hat{\vec{D}}' = c_0\nabla_a \times \hat{\vec{H}} \tag{11}$$

$$\vec{D}'(\omega) = \epsilon_r^*(w)\vec{E}'(\omega) \tag{12}$$

$$j\omega\hat{\vec{H}} = -c_0\nabla_a \times \hat{\vec{E}}' \tag{13}$$

where

$$\nabla_a \equiv \frac{1}{s_x}\,\partial_x\hat{a}_x + \frac{1}{s_y}\,\partial_x\hat{a}_y + \frac{1}{s_z}\,\partial_x\hat{a}_z \tag{14}$$

and

$$\left[\hat{U}_x \;\; \hat{U}_y \;\; \hat{U}_z\right]^T = [G]^{-1}[U_x \;\; U_y \;\; U_z]^T \tag{15}$$

with $\vec{U}$ one of $\vec{E}'$, $\vec{D}'$, $\vec{H}$ field vectors, and

$$[G] = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & s_z \end{bmatrix}. \tag{16}$$

It should be noted that the dielectric properties of the materials in the FDTD mesh (which can be lossy and frequency dependent) are accounted for in the $\epsilon_r^*(w)$ term which relates $\vec{D}'$ to $\vec{E}'$.

To provide enhanced evanescent wave absorption, s_x, s_y, s_z are chosen according to [6]

$$s_x = s_{x0}(x)\left[1 + \frac{\sigma_x(x)}{j\omega\epsilon_0}\right] \tag{17}$$

$$s_y = s_{y0}(y)\left[1 + \frac{\sigma_y(y)}{j\omega\epsilon_0}\right] \tag{18}$$

$$s_z = s_{z0}(z)\left[1 + \frac{\sigma_z(z)}{j\omega\epsilon_0}\right] \tag{19}$$

where the particular choice of unity values for $s_{x0}(x)$, $s_{y0}(y)$, and $s_{z0}(z)$ would lead to the traditional $D-E-H$ unsplit PML formulation [2].

For the purposes of terminating a waveguide, absorption in one direction is sufficient. Thus

$$s_x = s_{x0}(x)\left[1 + \frac{\sigma_x(x)}{j\omega\epsilon_0}\right] \tag{20}$$

$$s_y = 1 \tag{21}$$

$$s_z = 1. \tag{22}$$

The choice of $s_{x0}(x)$ and $\sigma_x(x)$ can be done as explained in [6].

The FDTD update equations for all of the components can therefore be derived as follows:

$$D_x^{n+1} = D_x^n + g_{i1}(ix)\Delta t\, curl\_zy + g_{i2}(ix)I_{Dx}^{n+1} \tag{23}$$

$$D_y^{n+1} = g_{i3}(ix)D_y^n + g_{i4}(ix)\Delta t\, curl\_xz \tag{24}$$

$$D_z^{n+1} = g_{i3}(ix)D_z^n + g_{i4}(ix)\Delta t\, curl\_yx \tag{25}$$

$$I_{Dx}^{n+1} = I_{Dx}^n + \Delta t\, curl\_zy \tag{26}$$

$$H_x^{n+1/2} = H_x^{n-1/2} + f_{i1}(ix)\Delta t\, curl\_yz + f_{i2}(ix)I_{Hx}^{n+1/2} \tag{27}$$

$$H_y^{n+1/2} = f_{i3}(ix)H_y^{n-1/2} + f_{i4}(ix)\Delta t\, curl\_zx \tag{28}$$

$$H_z^{n+1/2} = f_{i3}(ix)H_z^{n-1/2} + f_{i4}(ix)\Delta t\, curl\_xy \tag{29}$$

$$I_{Hx}^{n+1/2} = I_{Hx}^{n-1/2} + \Delta t\, curl\_yz \tag{30}$$

where ix indicates the spatial index in the x direction (direction of absorption) and

$$x_{n1} = s_{x0}(ix)$$

$$x_{n2} = s_{x0}(ix)\sigma(ix)\,\frac{\Delta t}{\epsilon_0}$$

$$g_{i1}(ix) = x_{n1}c_0$$

$$g_{i2}(ix) = x_{n2}c_0$$

$$g_{i3}(ix) = \frac{x_{n1} - 0.5x_{n2}}{x_{n1} + 0.5x_{n2}}$$

$$g_{i4}(ix) = \frac{c_0}{x_{n1} + 0.5x_{n2}}.$$

The curl terms $(curl\_xz, curl\_yx, curl\_zy, curl\_zx, curl\_xy, curl\_yz)$ in the update equations can be discretized as in the conventional FDTD technique.

III. Results and Comparison With Other Techniques

In order to validate the performance of the formulated unsplit PML for evanescent waves, a comparison is made with the regular unsplit PML and modal absorption [7]. For this purpose, a rectangular waveguide that extends in the x direction is terminated with the absorbing boundaries under test. The FDTD grid resolution is chosen to be $\lambda/40$ (where λ is the free space wavelength for the center frequency of the source) and the time step is chosen as 85% of the Courant number to ensure stability. A sinusoidally modulated Gaussian soft source (with a center frequency of 10 GHz and a double sided bandwidth of 10 GHz) is placed at the center of the waveguide (with y polarization), with the negative x side of the waveguide terminated with a thick 30 layer PML for almost perfect absorption and separated by 75 cells from the source so that the evanescent waves decay completely before reaching the boundary. Two different simulations are performed in order to extract the reflection from the test ABC as illustrated in Fig. 1. The first simulation is an infinite waveguide with the soft source in the middle and it is used for calculating the voltages that travel in the positive x direction at the plane of observation. The second simulation is similar to the first one, except that the test ABC is used to terminate the right side of the waveguide in close proximity of the plane of excitation (two FDTD cells). The distance between the observation plane and the test ABC is one FDTD cell.

Fig. 2 shows the reflection coefficient for the TE_{30} mode in a rectangular waveguide of dimensions 2 cm by 4 cm. The pro-

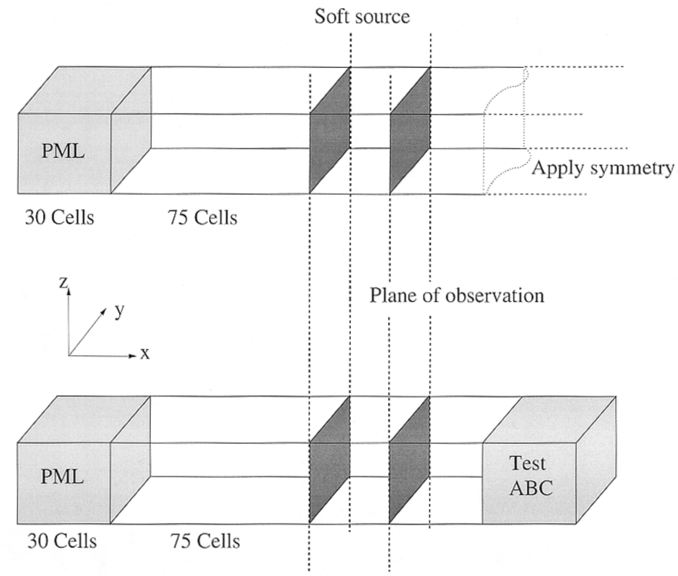

Fig. 1. Test scheme for comparing different ABCs.

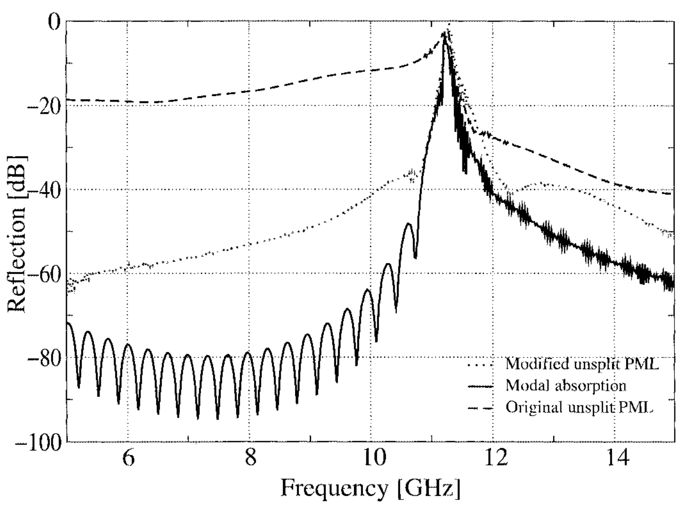

Fig. 2. Comparison of the reflection coefficient for the TE_{30} mode among different absorbing boundaries.

posed PML formulation shows an improvement of 20 to 40 dB in the absorption of evanescent mode as compared to the traditional unsplit $D-E-H$ formulation. Even though modal absorption is superior to the other two implementations of PML, it does require a prior knowledge of the modal fields which may be difficult to obtain in practical cases (as in inhomogeneous waveguides). The number of PML layers used for both tests was 12 and a reflection coefficient, R_{th}, of 10^{-6} with second order implementation was used for the modified unsplit PML.

To ensure a comparison based on best performance of the considered PML boundaries, the conductivities of the PML layers in the traditional $D-E-H$ unsplit formulation have been varied to obtain the optimal absorption. Thus, Fig. 2 shows the best absorption obtained with the original unsplit PML compared to the best absorption for the modified unsplit PML. No instabilities were observed in the simulations.

The modified unsplit PML retains the advantages of the regular unsplit PML while performing better in general for waveguide problems. The modified PML is once again independent of the real material parameters such as dielectric constants and conductivities and, unlike the formulation in [6], the real conductivities are not included in the E and H field updates.

IV. Conclusions

An unsplit $D-E-H$ PML formulation for the absorption of evanescent waves in waveguide problems is presented. It is shown that the proposed formulation noticeably outperforms the traditional $D-E-H$ unsplit PML formulation in the evanescent region by 20 to 40 dB while retaining all of its benefits and advantages originating from the independence of the PML conductivities from the dielectrics in the FDTD mesh.

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